PERMANENT MAGNET STABILITY: ADDENDUM

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Introduction

The object of this addendum is to give more details on the derivation by R. Street and J.C. Wooley and to suggest a simple experiment to rapidly and credibly quantify the long term stability of a permanent magnet. I reproduce here the essential of the derivation, with some minor corrections and changes in notation. Interested readers can refer to the complete paper, bearing in mind that the theory was developed in the context of a study of magnetic viscosity. This phenomenon, which is closely related to long-term decrease in magnetization refers to the fact that the magnetization does not follow the excitation instantaneously. In a magnetic viscosity experiment, an applied magnetic field is suddenly increased and one monitors the magnetization as a function of time. The magnetization typically reaches a "steady" value after periods on the order of 100 minutes.

As illustrated in Figure 1, hard ferrites are composed of small ferromagnetic regions or "grains" generally about 1 μ in size, closely packed and separated by non-ferromagnetic media. Each grain is constituted of a few domains [1] separated by walls pinned by various lattice imperfections. Thermal fluctuations can induce local strain variations and changes in the magnetic anisotropy constant resulting in wall nucleation and changes in the net magnetization of the grains. Regardless of the exact nature of the mechanism, the energy required to nucleate a wall can be regarded as the activation energy for the grain.

[1] Note that the situation is different in conventional materials such as Alnico where shape anisotropy prevails. In that case the grains are below critical size and are **single-domain**.

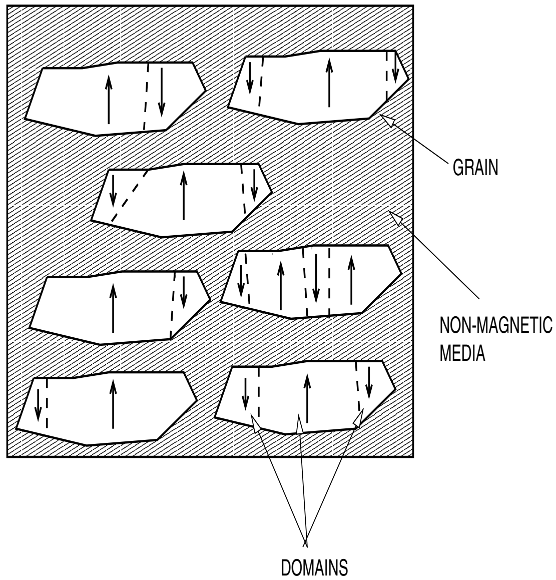

Figure 1: *Microstructure of a ferrite magnet. Note that both the orientation of the magnetization and the critical grain size are determined by magnetocrystalline rather than shape anisotropy.*

Street and Wooley's Theory

It is supposed that at some time $t = t_0$, a certain number of grains have their net magnetization vector in positions which can be described as "metastable".

Consider the number N of domains characterized by activation energies between E and $E + dE$ at the time t

$$N(E,t) = f(E,t)dE \tag{1}$$

The rate of change of N due to thermal activation a temperature T is

$$\frac{dN}{dt} = -Cf(E,t)\exp\left(-E/kT\right)dE \tag{2}$$

where C is a constant which depends on the material. Equation (2) is satisfied by

$$\begin{aligned} N(t) &= f(E,t_0)\exp\left[-\lambda(E)t\right]dE && (3)\\ &= f_0(E)\exp\left[-\lambda(E)t\right]dE && (4)\end{aligned}$$

where

$$\lambda(E) \equiv C\exp\left(-E/kT\right) \tag{5}$$

If each activation contributes an average amount m to the magnetization, then the activation of dN regions results in a mean **decrease** of the magnetization M

$$dM = -mCf_0(E)\exp -\lambda t \exp(-\frac{E}{kT})dEdt \quad (6)$$

Integrating over all values of the activation energy

$$\frac{dM}{dt} = -mC\int_{E_0}^{E_m} f_0(E)\exp -\lambda t \exp(-\frac{E}{kT})dE \quad (7)$$

In practice, the distribution $f_0(E)$ and the limits E_0, E_m are unknown. Three elementary cases are relevant:

1. $f_0(E) = p$ (a constant)
2. $f_0(E) = p$ for $0 < E < E_0$; 0 otherwise
3. $f_0(E) = N_0\delta(E - E_s)$

Case no 2 is obviously physically more reasonable. Case no 3 corresponds to the trivial case where all the grains have identical activation energies. Substituting

$$d\lambda = -\frac{C}{kT}\exp(-\frac{E}{kT})dE \quad (8)$$

and integrating over λ, one gets the following expressions.

Case no 1

$$\frac{dM}{dt} = -\frac{mpkT}{t}\left[1 - \exp(-Ct)\right] \quad (9)$$

If $Ct << 1$,

$$\Delta M = -mpkTC \cdot (t - t_0) \quad (10)$$

if $Ct >> 1$,

$$\Delta M = -mpkT\log(t/t_0) \quad (11)$$

Street and Wooley point out that which one of the above assumptions on the magnitude of Ct is more correct can be decided by experiment; in general it is found that magnetic viscosity is well described when ΔM is a linear function of $\log t$ and thus $Ct >> 1$. In a context where the activation energy model is applied to the long-term stability (20 years) of magnetization, there is little doubt that $Ct >> 1$ is the correct assumption.

Note that according to (11) $\Delta M(\infty) = -\infty$ which is obviously unphysical. This result is the consequence of the assumption $f(E) = p$ which is impossible unless $p \to 0$, since one must have

$$\int_0^\infty f_0(E)dE = N_0 \quad (12)$$

Case no 2

$$\frac{dM}{dt} = \frac{mpkT}{t}\left[\exp(-\lambda_0 t) - \exp(-Ct)\right] \tag{13}$$

Equation (11) can be put in the form

$$dM \simeq -mpkT \int_{t_C}^{t} \frac{1}{t}\exp(-\lambda_0 t)dt + M_C \tag{14}$$

where $\lambda_0 \equiv \lambda(E_0)$ and M_C is the value of ΔM after the term $\exp(-Ct)$ becomes negligible with respect to $\exp(-\lambda_0 t)$. Substituting the expansion

$$\frac{1}{t}\exp(-\lambda_0 t) \simeq 1/t - \lambda_0 \tag{15}$$

into (14), one obtains

$$\Delta M = -mpkT\left[\log t/t_C - \lambda_0(t - t_C)\right] \tag{16}$$

Note that (16) is only approximate; for $t \to \infty$, the integral (14) converges to a **finite** value. Note also the sign of the linear correction. Expressing p in terms of the total number of grains and the width of the activation energy density function,

$$p = \frac{N_0}{(E_m - E_0)} \tag{17}$$

one can recast (16) under the more physically suggestive form

$$\Delta M = -mN_0\frac{kT}{(E_m - E_0)}\left[\log(t/t_C) - \lambda_0(t - t_C)\right] \tag{18}$$

Case no 3

$$\frac{dM}{dt} = -mN_0\lambda_s\exp(-\lambda_s t) \tag{19}$$

where

$$\lambda_s \equiv \exp(-E_s/kT) \tag{20}$$

and N_0 is the total number of grains with average magnetization m at time $t = t_0$. Integrating

$$\Delta M = -mN_0\left[1 - \exp(-\lambda_s t)\right] \tag{21}$$

Once again, note that this expression predicts that $M(t = \infty) = 0$.

A Suggested Experiment

Long term measurements of magnet stability are difficult to make: in the context of the recycler ring, we are interested in decrease in magnetization on the order of a fraction of 1% over 10 years. An interesting way to **quickly** and **credibly** estimate the long term stability of a ferrite magnet would be to perform a magnetic viscosity experiment. To avoid the magnetic excitation to irreversibly perturb the state of magnetization of the sample, one would most likely have to limit the amplitude of the excitation. The principle is simple: a sudden change in the excitation induces a change of state for a much larger number of magnetic grains than a a small temperature fluctuation.

References

[1] Street R. and Wooley, J.C., A Study of Magnetic Viscosity, *Proc. Phys. Soc.* **A62***, 562-572, (1949)*